The Category of Iterative Sets in Cubical Agda

Stockholm–Gothenburg Type Theory Meeting

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1. Motivation

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Previous article by Gratzer, Gylterud, Mörtberg, Stenholm [1]

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Idea: Easier to formalize in cubical setting

2. Iterative Sets

Iterative Multisets

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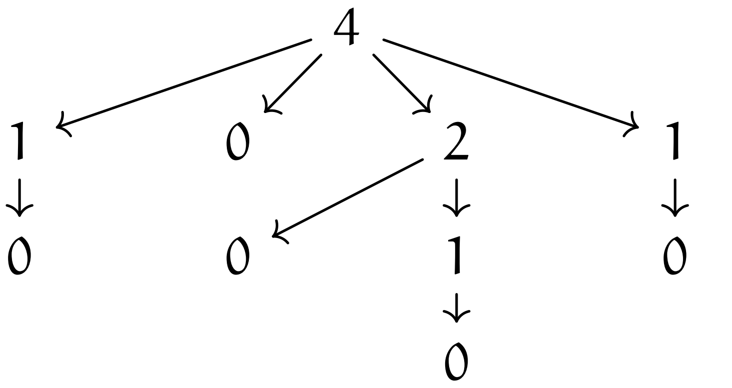

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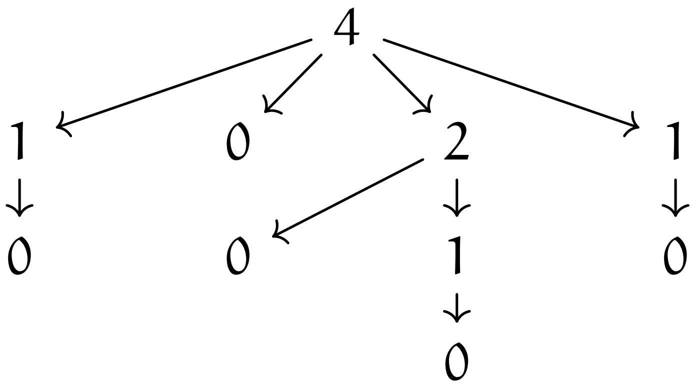

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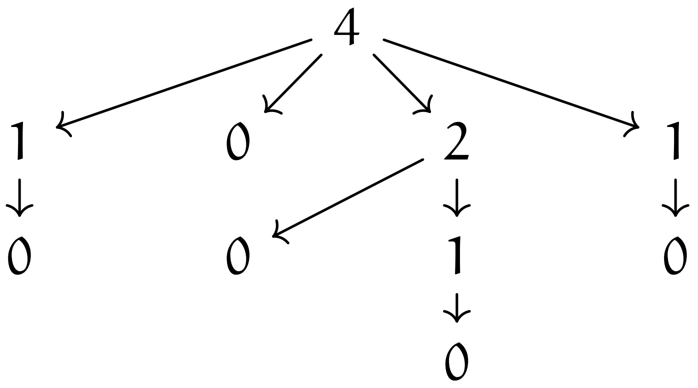

Data: some **type** A and some **function** $f : A \to \ldots$

Definition **W-types:** $W_{x:A} B(x)$ with $A : \mathrm{Type}$ and $B : A \to \mathrm{Type}$ has elements

$$\mathrm{sup}(a, f) \quad \textbf{with} \quad a : A, \quad f : B(a) \to W_{x:A} B(x)$$

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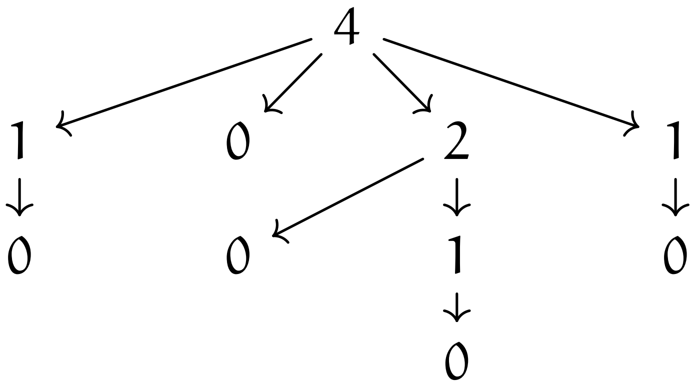

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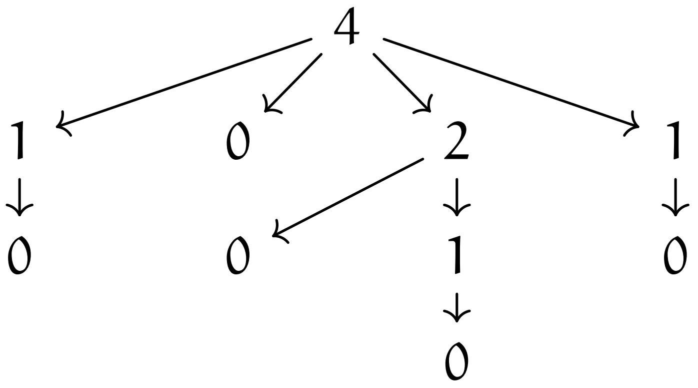

- **multisets** with **multiplicities** $(x \in^\infty (\mathsf{sup}(B, g)) :\equiv \mathsf{fiber}_g(x) :\equiv \sum_{b:B} g(b) =_{V^\infty} x$

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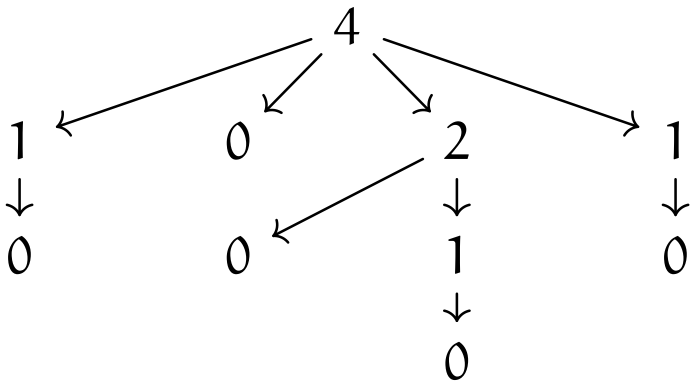

- **multisets** with **multiplicities** $(x \in^\infty (\sup(B, g)) := \mathrm{fiber}_g(x) := \sum_{b:B} g(b) \equiv_{V^\infty} x$
- **decoding:** $\mathrm{El}(\sup(A, f)) := A$

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- **Fibration Identity Principle** (~ Extensionality)

$$(x \equiv_{V^0} y) \simeq \prod_{z:V^0} \left(z \in^0 x\right) \simeq \left(z \in^0 y\right)$$

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3. Iterative Sets as a model of Dependent Type Theory

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- a **functor** $- \,.\, - : \int \mathrm{Ty} \to \mathcal{C}$
- for each $(\Gamma, A) : \int \mathrm{Ty}$ a **natural equivalence** in $\Delta : \mathcal{C}$:

$$\mathcal{C}(\Delta, \Gamma \,.\, A) \simeq \sum_{\gamma : \mathcal{C}(\Delta, \Gamma)} \mathrm{Tm}(\Delta, \gamma^* A)$$

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$$\mathrm{Tm}(\Gamma, S_\Gamma(A, B)) \simeq \sum_{a : \mathrm{Tm}(\Gamma, A)} \mathrm{Tm}\big(\Gamma, \langle 1_\Gamma, a\rangle^* B\big)$$

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4. Formalization

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  2. "Cubical"
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- **General statement** for all Tarski set-universes of sets

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- $\rightsquigarrow$ **Π / Σ types**

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Naturality of Context Extension

$$
\begin{array}{ccccc}
\Delta & & \mathcal{C}(\Delta, \Gamma \,.\, A) & \xrightarrow{\;\cong\;} & \sum_{\tau : \mathcal{C}(\Delta,\Gamma)} \mathrm{Tm}(\Delta, (\tau^* A)) \\
 & & & & \Big\downarrow {\scriptstyle (\mathrm{id}, -[\sigma])} \\
 & & & & \sum_{\tau : \mathcal{C}(\Delta,\Gamma)} \mathrm{Tm}(\Delta', (\sigma^*(\tau^* A))) \\
\Big\uparrow {\scriptstyle \sigma} & & \Big\downarrow {\scriptstyle \sigma^*} & & \Big\| {\scriptstyle (\mathrm{id}, \mathrm{transport}_p(-))} \\
 & & & & \sum_{\tau : \mathcal{C}(\Delta,\Gamma)} \mathrm{Tm}\big(\Delta', \big((\tau \circ \sigma)^* A\big)\big) \\
 & & & & \Big\downarrow {\scriptstyle (\sigma^*, \mathrm{id})} \\
\Delta' & & \mathcal{C}(\Delta', \Gamma \,.\, A) & \xrightarrow{\;\cong\;} & \sum_{\tau : \mathcal{C}(\Delta',\Gamma)} \mathrm{Tm}(\Delta', (\tau^* A))
\end{array}
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\Gamma & & \mathrm{Tm}(\Gamma, S_\Gamma(A, B)) & \xrightarrow{\ \cong\ } & \sum_{a' : \mathrm{Tm}(\Gamma, A)} \mathrm{Tm}\big(\Gamma, \langle 1_\Gamma, a' \rangle^* B\big) \\
\uparrow {\scriptstyle f} & & \downarrow {\scriptstyle -[f]} & & \downarrow {\scriptstyle (\mathrm{id},\, -[f])} \\
 & & & & \sum_{a' : \mathrm{Tm}(\Gamma, A)} \mathrm{Tm}\big(\Delta, f^* \langle 1_\Gamma, a' \rangle^* B\big) \\
 & & & & \downarrow {\scriptstyle \left(\mathrm{id},\, \mathrm{subst}_q^{\mathrm{Tm}(\Delta, -)}(-)\right)} \\
\Delta & & \mathrm{Tm}(\Delta, f^* S_\Gamma(A, B)) & & \sum_{a' : \mathrm{Tm}(\Gamma, A)} \mathrm{Tm}\big(\Delta, \langle 1_\Delta, a'[f] \rangle^* \langle f, A \rangle^* B\big) \\
\| & & \downarrow {\scriptstyle \mathrm{subst}_p^{\mathrm{Tm}(\Delta, -)}(-)} & & \downarrow {\scriptstyle (-[f],\, \mathrm{id})} \\
\Delta & & \mathrm{Tm}\big(\Delta, S_\Delta\big(f^* A, \langle f, A \rangle^* B\big)\big) & \xrightarrow{\ \cong\ } & \sum_{a : \mathrm{Tm}(\Delta, f^* A)} \mathrm{Tm}\big(\Delta, \langle 1_\Delta, a \rangle^* \langle f, A \rangle^* B\big)
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Bibliography

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